MELT CONTROL: CHARGE OPTIMIZATION VIA STOCHASTIC PROGRAMMING

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Abstract

This paper introduces melt control as a good case for application of two- and multistage stochastic programming models. Sources of uncertainties are described and several methods of input generation are presented. The implementation based on real data compares decisions and costs obtained by solving stochastic programs with different numbers of stages and a different structure of the scenario tree. The results give a clear evidence in favor of the proposed stochastic programming methodology.

1 Motivation

Melt control problems belong to the broad field of production control applications. They are studied as one of the production steps in iron and steel works. Melt control problems may be fully separated from other foundry optimization problems, which simplifies the model building and its solution. Their importance arises because foundries usually have high overheads, and hence, even small percentual savings may recover a significant amount of money. In addition, material inputs represent the biggest part of the total melting costs.

The produced alloys and input materials are composed of certain basic elements (iron, carbon, etc.). The production process consists of several steps (e.g., charge, alloying). During the alloying process, the hot melt in the furnace is enriched with certain input materials (return materials, scrap, ferroalloys, etc.) and a new mixture is melted again. Hence, the problem has a natural multistage decision structure. Whereas the unit costs of the inputs are known at the time of decision making, the composition of input materials is not known precisely and it was modeled as random already in Evers (1967). In each step of the process, the melt composition changes and particularly, random losses of elements in the melt must be considered. During heating of the melt the amounts of elements change randomly, e.g., due to arise of slag and oxidation. These losses depend on the

composition of the melted materials. In some cases, they may be influenced also by the used amounts of these materials, which will not be considered here. The remaining amount of an element is expressed as a linear function in the input quantities of all considered elements, the coefficients are called *utilizations* of the considered element related to the amount of other elements in the melt.

The goal is to find amounts of the input materials in the lowest cost so that the prescribed output alloy composition is achieved. We use scenario-based two- and three-stage stochastic linear programs to illustrate basic modeling ideas for charge optimization of induced and electric-arc furnaces. For a general approach to melt control, developed for any alloy, furnace, and technology see Popela (1998a, 1998b).

2 Examples

In the following two simplified examples, the random losses, and hence the related utilizations of elements are taken as the only random variables. This assumption can be accepted for example for melting ferroalloys of a guaranteed composition and it will be relaxed in the last example. Historical melt reports are available and may be used to construct scenarios or scenario trees of utilizations for the melt control problems; see Section 3.

2.1 Two-Stage Induced Furnace Charge Optimization

We begin with a simple model for charge optimization of iron production in an induced furnace – a model with a common two-stage structure: Through the initial charge decision the final cost of the melt is minimized taking into account also the consequences of possible random losses and the requirements on the final composition of the melt. The problem in extensive form (see Birge and Louveaux (1997)) is

$$\text{minimize } \sum_{j\in J_1} c_j x_j^1 \; + \; \sum_{k_2\in\mathcal{K}_2} p_{k_2} \sum_{j\in J_2} c_j x_j^{k_2} \tag{1}$$

subject to

$$l_{i1} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^E \sum_{j\in J_1} a_{lj} x_j^1 \le\; u_{i1},\; i=1,\ldots,m_1 \tag{2}$$

$$l_{i2} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j\in J_1} a_{lj} x_j^1 + \sum_{j\in J_2} a_{ij} x_j^{k_2} \le\; u_{i2},\; i=1,\ldots,m_2,\; k_2\in\mathcal{K}_2 \tag{3}$$

$$x_j^1 \ge 0,\; j\in J_1, \quad x_j^{k_2}\ge 0,\; j\in J_2,\; k_2\in\mathcal{K}_2 \tag{4}$$

where $t=1,2$ are stages,
J_t is the set of indices of input materials available at stage t,
m_t is the number of elements at stage t and indices i and l specify them,
$c_j \ge 0$, $j\in J_t$ are known unit costs of jth input material,

$l_{it}, u_{it} \geq 0$ are prescribed lower and upper goal bounds for the amount of the ith element in melt composition at stage t,
$a_{ij} \geq 0, \sum_i a_{ij} \leq 1 \, \forall j$ denote the amount of ith element in the unit amount of jth input material,
$x_j^1 \geq 0$ denote the first-stage decision variables, the amount of jth input material at the beginning of the melt process (charge),
$x_j^{k_2}$, $k_2 \in \mathcal{K}_2$ denote the second-stage decision variables, which stay for the additional amount of jth input material assigned under scenario k_2 (alloying).

The only random elements are utilizations. Let
$\tau_{il}^{k_2}$ be the utilization of ith element related to the amount of lth element in the melt when scenario k_2 occurs, $0 \leq \tau_{il}^{k_2} \leq 1$; k_2 are indices of scenarios with probabilities $p_{k_2} \geq 0$. The frequently considered case $\tau_{il}^{k_2} = 0$ when $i \neq l$ means that interactions of random losses are ignored.

In the first-stage constraints (2), τ_{il}^E stands for an experience-based 'standard' utilization which applies to the first stage. Usually, $\tau_{il}^E = \sum_{k_2 \in \mathcal{K}_2} p_{k_2} \tau_{il}^{k_2} \; \forall i, l$, i.e. the average utilizations. These constraints reflect the metallurgical rules which aim at the process control stability and in general, they cannot be neglected. On the other hand, possible losses of the materials added in the second stage of the melting process are negligible and are not considered.

The model has a common two-stage structure with fixed recourse. The charge decisions x_j^1 take into account consequences of the random utilizations at the second stage, making the final alloy composition reachable.

2.2 Three-Stage Electric-Arc Furnace Charge Optimization

The situation is more complicated with a steel production in an electric-arc furnace. Because of two alloying phases, the whole process must be modeled as a three-stage problem. To simplify the model description, we mostly utilize the notation of Example 2.1 and the arborescent form introduced, e.g. in Birge and Louveaux (1996) or in Part II of Dupačová et al. (2002), with indices k_t corresponding to nodes of the tth stage (i.e., to stage t scenarios) $t = 2, 3$, and $a(k_t)$ denoting the index of the ancestor node to the node k_t.

To obtain the Markovian structure of the model constraints, the melt composition is described explicitly by additional auxiliary variables $h_i^{k_t}$ describing the state of the decision process – the amount of ith melt element at node k_t of stage t before a subsequent decision was taken. We assume an empty furnace at the beginning of the process, hence, $h_i^1 = 0 \, \forall i$ at stage 1, then $h_i^{k_2} = \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j \in J_1} a_{lj} x_j^1$, etc. Hence, for $t > 1$ and all elements considered at stage t,

$$h_i^{k_t} = \sum_{l=1}^{m_{t-1}} \tau_{il}^{k_t} \left(h_l^{a(k_t)} + \sum_{j \in J_{t-1}} a_{lj} x_j^{a(k_t)} \right). \tag{5}$$

The model is

$$\text{minimize } \sum_{j\in J_1} c_j x_j^1 \; + \; \sum_{t=2}^{3} \sum_{k_t\in\mathcal{K}_t} p_{k_t} \sum_{j\in J_t} c_j x_j^{k_t} \tag{6}$$

subject to

$$\sum_{l=1}^{m_{t-1}} \tau_{il}^{k_t} \left(h_l^{a(k_t)} + \sum_{j\in J_{t-1}} a_{lj} x_j^{a(k_t)} \right) - h_i^{k_t} = \; 0,\; i = 1,\ldots,m_{t-1},\; k_t \in \mathcal{K}_t,\; t = 2,3 \tag{7}$$

$$l_{it-1} \;\le \sum_{k_t\in\mathcal{K}_t} p_{k_t} h_i^{k_t} \le \; u_{it-1},\; i = 1,\ldots,m_{t-1},\; t = 2,3 \tag{8}$$

$$l_{i3} \;\le h_i^{k_3} + \sum_{j\in J_3} a_{ij} x_j^{k_3} \;\le u_{i3},\; i = 1,\ldots,m_3 \tag{9}$$

$$x_j^1 \;\ge\; 0,\; j \in J_1,\; x_j^{k_t} \;\ge\; 0,\; k_t \in \mathcal{K}_t,\; j \in J_t,\; t = 2,3. \tag{10}$$

The expected cost of melt is minimized subject to constraints requiring that during the whole melt process the average melt composition satisfies the given bounds and that the final product satisfies these bounds for all scenarios. In the last stage, full utilization of added materials is assumed. For $t = 2$, the constraints (8) are in agreement with (2) based on average utilizations $\tau_{il}^E = \sum_{k_2\in\mathcal{K}_2} p_{k_2} \tau_{il}^{k_2}$.

An extension to more that three stages is evident. However, it is clear that the two introduced models must be further generalized and significantly extended (e.g., involving additional linear technological and inventory constraints, uncertain scrap composition or for more than three stages) before they become applicable in real-world foundries; see Popela (1998a, 1998b) for a discussion and suggestions.

2.3 Random input composition

Theoretically, the probability distribution of the jth input composition $a_{ij}\,\forall j$ may be estimated from data obtained by a repeated chemical analysis of the input. In practise, normative values (based on such measurements) are used and/or intervals $\underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}$ are built separately for each of input materials. This information is exploited in the scenario generation procedure which will be explained in Section 3 and which provides scenarios $s \in \mathcal{S}$ of input matrices $\boldsymbol{A} = (a_{ij}, i = 1,\ldots,m_1, j \in J_1)$ and their probabilities π_s; we index by superscripts s the corresponding elements of matrices $\boldsymbol{A}$ and the second stage variables. These scenarios and their probabilities are supposed to be known before the melting process starts and the first-stage decisions depend on this probabilistic specification. In addition, suppose that the composition of the (high quality) input added in the second stage (alloying) is known and that the initial charge is based on expert estimates $a_{ij}^E, i = 1,\ldots,m_1$, of the composition of input materials $j \in J_1$. Assuming independence of the random input composition and utilizations we rewrite Example 2.1 as follows:

$$\text{minimize } \sum_{j\in J_1} c_j x_j^1 \; + \; \sum_{s\in\mathcal{S}} \sum_{k_2\in\mathcal{K}_2} p_{k_2}\pi_s \sum_{j_2\in J_2} c_{j_2} x_{j_2}^{k_2 s} \tag{11}$$

subject to

$$l_{i1} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^{E} \sum_{j\in J_1} a_{lj}^{E} x_j^1 \;\le\; u_{i1},\; i=1,\ldots,m_1 \tag{12}$$

$$l_{i2} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j\in J_1} a_{lj}^{s} x_j^1 + \sum_{j\in J_2} a_{ij} x_j^{k_2 s} \;\le\; u_{i2},\; i=1,\ldots,m_2,\; k_2\in\mathcal{K}_2,\; s\in\mathcal{S} \tag{13}$$

$$x_j^1 \ge 0,\; j\in J_1, \quad x_{j_2}^{k_2 s} \ge 0,\; j_2\in J_2,\; k_2\in\mathcal{K}_2,\; s\in\mathcal{S}. \tag{14}$$

3 Scenario generation

When building a melt control program, scenario generation is one of the most important tasks. There is a large amount of melt control reports, however, these reports contain only an indirect information about scenarios. Usually only measurements $h_i^{k_t}\,\forall i,k_t$ and the inputs $x_j^{k_t}\,\forall j,k_t$ are specified together with a full or partial information about the composition $\boldsymbol{A} := (a_{ij},\forall i,j)$ of the input materials. Although the ideas are also valid for multistage problems, we will explain them for the simple two-stage melt control program formulated in Example 2.1. In this case the melt control reports list most of all the inputs amounts $x_j^1,\; j\in J_1$ and the resulting melt composition

$$h_i^{k_2} = \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j\in J_1} a_{lj} x_j^1,\; i=1,\ldots,m_1,\; k_2\in\mathcal{K}_2. \tag{15}$$

3.1 Scenarios of diagonal utilization matrices

In the simplest case, with diagonal utilization matrices $\boldsymbol{T}$ and a known composition $\boldsymbol{A}$ of input materials, the nonzero utilizations $\tau_{ii}^{k_2}$ are obtained as the solution of the trivial system of equations

$$h_i^{k_2} = \tau_{ii}^{k_2} \sum_{j\in J_1} a_{ij} x_j^1,\; i=1,\ldots,m_1,\; k_2\in\mathcal{K}_2. \tag{16}$$

Utilizations obtained in this way may be used directly as *measurement-based* scenarios in the second-stage constraints

$$l_{i2} \;\le\; \tau_{ii}^{k_2} \sum_{j\in J_1} a_{lj} x_j^1 + \sum_{j\in J_2} a_{ij} x_j^{k_2} \;\le\; u_{i2},\; i=1,\ldots,m_2,\; k_2\in\mathcal{K}_2$$

compare with (3).

The melt control reports contain measurements on the auxiliary state variables and report the applied decisions for all stages of the production process: The tth stage related information is composed from the initial measurement $h_i^{k_{t-1}}$, stage related inputs $x_j^{k_t}$ and the final measurement $h_i^{k_t}$. Assuming an empty furnace at the beginning, the result is a "fan" of measurement-based scenarios of utilizations $(\tau_{ii}^{k_1},\ldots,\tau_{ii}^{k_T})\,\forall i$ which branch only at the root and have equal probabilities.

Another possibility is to generate a limited number, say K, of diagonal utilization matrices taking into account specific statistical properties of marginal probability distributions of their diagonal elements $\tau_{ii}\,\forall i$, e.g., their expectations μ_i and variances σ_i^2, and also covariances ρ_{il} of couples τ_{ii}, τ_{ll}. This means to estimate these moment values, e.g., using the past melt reports, and to find a feasible solution $\boldsymbol{T}^k, p_k,\ k = 1, \ldots, K$ of the *moment fitting* problem

$$\sum_{k=1}^{K} p_k \tau_{ii}^k = \mu_i,\ i = 1, \ldots, m_1,$$

$$\sum_{k=1}^{K} p_k (\tau_{ii}^k - \mu_i)^2 = \sigma_i^2,\ i = 1, \ldots, m_1,$$

$$\sum_{k=1}^{K} p_k (\tau_{ii}^k - \mu_i)(\tau_{ll}^k - \mu_l) = \rho_{il},\ i, l = 1, \ldots, m_1,$$

$$\sum_{k=1}^{K} p_k = 1, p_k \geq 0,\ k = 1, \ldots, K.$$

To get the *fitted scenarios* and their probabilities means to solve this nonlinear system with respect to p_k and τ_{ii}^k, $\forall i, k$. The system can be further extended for constraints on the ranges of the utilization values, on higher order moments, etc. For a consistent statistical specification, the general results of the moment problem imply that a solution of such system exists for a modest number of scenarios. Otherwise one may try to get an approximate solution, e.g., by minimizing the weighted sum of squares of differences

$$\sum_{i=1}^{m_1}[\alpha_i(\sum_{k=1}^{K} p_k \tau_{ii}^k - \mu_i)^2 + \beta_i(\sum_{k=1}^{K} p_k (\tau_{ii}^k - \mu_i)^2 - \sigma_i^2)^2] + \sum_{i,l=1}^{m_1} \gamma_{il}(\sum_{k=1}^{K} p_k (\tau_{ii}^k - \mu_i)(\tau_{ll}^k - \mu_l) - \rho_{il})^2 \tag{17}$$

subject to $\sum_{k=1}^{K} p_k = 1, p_k \geq 0,\ k = 1, \ldots, K$. Parameters $\alpha_i, \beta_i, \gamma_{il}$ can be used to reflect importance and quality of data; see Dupačová et al. (2000) and Høyland and Wallace (2001) for a detailed discussion.

The assumption of diagonal utilization matrices facilitates scenario generation for the three– and multistage models, it is frequently used in practise and will be accepted also in the numerical illustration in Section 4. Still, an extension to general utilization matrices and to random composition of input material is important. System (15) cannot identify the utilizations in a unique way even if there is an experience based benchmark for their values, expressed for instance in the form of simple box constraints, lower and upper bounds on $\tau_{il}^{k_2}$ valid for all k_2. Allowing for random coefficients a_{lj}, independent of utilizations may help, see Subsection 3.4.

3.2 Scenario tree generation

The path-wise input by scenarios as described above does not display the information structure given by the technological process and a scenario tree should be

built. The number of branching points is linked with the stages of corresponding production process, similarly as in Example 2.2.

A simple case of a scenario tree refers to the *interstage independence* with tth stage utilizations $\tau_{ii}^{k_t}$ independent of utilizations in the preceding stages. This means that all melt reports concerning stage t may be used to get utilizations $\tau_{ii}^{k_t}$ for all nodes identified by $\tau_{ii}^{k_{t-1}}$. This approach carries over the equal scenario probabilities at all stages so that the probabilities of all paths from the root to leaves of the scenario tree are equal, too.

Accepting interstage independence means a simplification whose disadvantage is that the number of nodes of such tree grows rapidly. There are other ways to construct the scenario tree, see e.g. Dupačová et al. (2000, 2002). In Section 4, we apply the moment fitting approach by Høyland and Wallace (2001) explained briefly in Subsection 3.1 to generate a tree which mimics the statistical properties of the joint probability distribution, including the interstage dependence.

3.3 Scenarios of input composition

It is natural to build independent scenarios of composition for individual input materials. On the other hand, statistical dependence of the contents of the considered elements in the given input material should not be, in general, disregarded.

As the first possibility assume that the composition of the jth input material, $\boldsymbol{a}^j := (a_{ij}\,\forall i)$ is a multinormal vector, $\mathcal{N}(\boldsymbol{a}^{jE}, \boldsymbol{V}^j)$, with expectations $a_{ij}^E\,\forall i$ and the variance matrix estimated from experimental data obtained by chemical analysis. Scenarios $\boldsymbol{a}^{js}$ may be then sampled from this distribution. Another possibility is to sample scenarios from one-dimensional marginal distributions $\mathcal{N}(a_{ij}^E, \sigma_{ij}^2)$ or to discretize these marginal distributions independently for all i and to accept all m_1-tuples of these independent marginal scenarios as scenarios of $\boldsymbol{a}^j$. The next step is to select representative scenarios and their probabilities so that the moments values $\boldsymbol{a}^{jE}$, $\boldsymbol{V}^j$ are retained; see, e.g. Cariño et al. (1998) or Høyland and Wallace (2001).

If the intervals $\underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}$ listed separately for each element i provide the only available information we may accept the corresponding uniform distributions as the model of the random composition and to approximate these distributions by the nearest (in the sense of a selected probability metric) discrete uniform one-dimensional distributions. Such approximation depends on the chosen probability metric.

Let $F(t)$ denote the distribution function of a random variable and $\hat{F}(t)$ the distribution function obtained by an approximation. For instance, think of distributions carried by three scenarios $a_{ij}^E - \delta_{ij}$, a_{ij}^E, $a_{ij}^E + \delta_{ij}$, with equal probabilities 1/3. The optimal values of δ_{ij} are equal to $1/3(\overline{a}_{ij} - a_{ij}^E)$ for the Kolmogorov metric

$$\varrho_K(F, \hat{F}) = \sup_t |F(t) - \hat{F}(t)|$$

and to $2/3(\overline{a}_{ij} - a^E_{ij})$ for the Wasserstein metric

$$\varrho_W(F, \hat{F}) = \int |F(t) - \hat{F}(t)|dt.$$

Another recommendation is to put expectations $a^E_{ij} = 1/2(\underline{a}_{ij} + \overline{a}_{ij})$ and the $\sigma^2_{ij} = \frac{1}{16}(\underline{a}_{ij} - \overline{a}_{ij})^2$, to accept normal marginal distributions with these parameters and to approximate them by symmetric discrete probability distributions concentrated, say, again in three atoms, $a^E_{ij} - \delta_{ij}$, a^E_{ij}, $a^E_{ij} + \delta_{ij}$, with equal probabilities 1/3. This time the optimal values of δ_{ij} obtained by minimization of the Wasserstein metric are equal to $1.225\sigma_{ij}$, i.e. to $0.612(\overline{a}_{ij} - a^E_{ij})$. See Pflug (2001) for further examples of approximations by discrete distributions.

Concerning covariances, there are typically no records and only experts' knowledge may be used. Hence, once more, one may accept all m_1-tuples of the independent marginal scenarios obtained by discretization and select representative scenarios and their probabilities to fit the moments values $\boldsymbol{a}^{jE}$, $\boldsymbol{V}^j$.

Finally, observe that the random coefficients appear only on the "right-hand" sides of constraints (13) that may be rewritten as

$$l_{i2} - \sum_{l=1}^{m_1} \tau_{il} \sum_{j \in J_1} a_{lj} x^1_j \leq \sum_{j \in J_2} a_{ij} x^2_j \leq u_{i2} - \sum_{l=1}^{m_1} \tau_{il} \sum_{j \in J_1} a_{lj} x^1_j,\ i = 1, \ldots, m_1. \qquad (18)$$

Hence for a *given* charge x^1_j, $j \in J_1$, observed utilizations $\tau_{il}\, \forall i$, l and compositions $\boldsymbol{a}^j$, $j \in J_1$ the minimum cost additional input x^2_j, $j \in J_2$ solves the second-stage linear program

$$\min_{x^2_j \geq 0 \forall j} \sum_{j \in J_2} c_j x^2_j \quad \text{subject to (18)}$$

whose optimal value is a convex function of $h_i := \sum_{l=1}^{m_1} \tau_{il} \sum_{j \in J_1} a_{lj} x^1_j\, \forall i$. For $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ with expectation $a^E_{ij} = 1/2(\underline{a}_{ij} + \overline{a}_{ij})$ we have

$$h_i \in I_i := [\underline{h}_i, \overline{h}_i] \text{ and } h^E_i = 1/2(\underline{h}_i + \overline{h}_i) \qquad (19)$$

where $\underline{h}_i = \sum_{l=1}^{m_1} \tau_{il} \sum_{j \in J_1} \underline{a}_{lj} x^1_j\ \forall i$ and $\overline{h}_i = \sum_{l=1}^{m_1} \tau_{il} \sum_{j \in J_1} \overline{a}_{lj} x^1_j\ \forall i$. Hence, the *minimum expected cost of the additional input is attained for the average values* h^E_i by Jensen's inequality, i.e., by solving program (1)–(4) with $a_{ij} = a^E_{ij}\, \forall i$, $j \in J_1$ in (3). In this optimistic case, one uses the most favorable, degenerated distribution of state variables implied by the assumed intervals and expectations of the random composition $a_{ij} \forall i$, $j \in J_1$ of the input materials. The pessimistic, worst-case distribution corresponding to (19) is discrete, concentrated on vertices of the Cartesian product of intervals $I_i\, \forall i$, cf. Edmundson-Madansky bound. In the multi-dimensional case, an explicit formula may be given only under special assumptions such as the stochastic independence or separability of the second-stage optimal value with respect to h_i, $\forall i$ (see Birge and Louveaux (1)) which is not realistic in our context. One obtains then the worst-case probability distribution for which the marginal distributions of $h_i \forall$ are concentrated on $\underline{h}_i$, $\overline{h}_i$ with equal

probabilities 1/2. As a consequence (compare with (13)), for each $j \in J_2$ there are nonnegative second-stage variables $\underline{x}_j^{k_2}$ and $\overline{x}_j^{k_2}$ $\forall k_2$ and inequalities (3) split into

$$l_{i2} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j \in J_1} \underline{a}_{lj} x_j^1 + \sum_{j \in J_2} a_{ij} \underline{x}_j^{k_2} \;\le\; u_{i2},\; i = 1, \ldots, m_2,\; k_2 \in \mathcal{K}_2$$

and

$$l_{i2} \;\le\; \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j \in J_1} \overline{a}_{lj} x_j^1 + \sum_{j \in J_2} a_{ij} \overline{x}_j^{k_2} \;\le\; u_{i2},\; i = 1, \ldots, m_2,\; k_2 \in \mathcal{K}_2.$$

In the objective function (1), $x_j^{k_2}$ is replaced by the average $1/2(\underline{x}_j^{k_2} + \overline{x}_j^{k_2})$ $\forall k_2 \in \mathcal{K}_2$, $j \in J_2$. This is in agreement with (11).

An additional assumption of *unimodal* marginal distributions of h_i separately for each i results in one-dimensional worst-case marginal distributions *uniform* over the intervals I_i. This conclusion and other extensions may be found among results of the minimax approach and moment problems; see, e.g., Dupačová (1987).

Scenarios of matrices of the input composition may be then created by combining all possibilities taken into account for individual input materials:

Let s_j, $s_j \in \mathcal{S}_j$ be scenarios representing the random composition of the jth input and π_{s_j} their probabilities. Combining all possible outcomes for each of input materials leads to $S = \prod_{j \in J_1} (\#\mathcal{S}_j)$ scenarios of the technological matrices $\boldsymbol{A} = (a_{ij})$. Their probabilities π_s are equal to the product of the corresponding probabilities π_{s_j} $\forall j$.

3.4 Scenarios of non-diagonal utilization matrices

Assume that *independently* of utilizations the composition of individual inputs is indicated in the melt reports. To simplify the notation we index scenarios of the matrices $\boldsymbol{A} = (a_{ij})$ by a superscript s similarly as in Example 2.3. Assume that an expert is able to select groups of melting reports, say $\mathcal{S}(k_2)$, for which the same utilizations $\tau_{il}^{k_2}$ are likely. It means that for each k_2 *separately* utilizations $\tau_{il}^{k_2}$ satisfy the system

$$h_i^{k_2 s} = \sum_{l=1}^{m_1} \tau_{il}^{k_2} \sum_{j \in J_1} a_{lj}^s x_j^{1s},\; i = 1, \ldots, m_1,\; s \in \mathcal{S}(k_2). \tag{20}$$

The x_j^{1s}, $h_i^{k_2 s}$, a_{lj}^s $\forall i, j$ and $s \in \mathcal{S}(k_2)$ in (20) are known coefficients based on melting reports and the linear model (20) may be used to estimate utilizations $\tau_{il}^{k_2}$ $\forall i, l$. The probability of the resulting scenario – the (non-diagonal) matrix $\boldsymbol{T}^{k_2}$ – is proportinal to the cardinality $\#\mathcal{S}(k_2)$ of $\mathcal{S}(k_2)$.

If $\#\mathcal{S}(k_2)$ is large in comparison with m_1, the set $\mathcal{S}(k_2)$ may be replaced by a union of disjoint sets $\mathcal{S}'(k_2)$ each consisting of of m_1 elements such that the matrix of the corresponding subsystem of (20) is non-singular. By solving these subsystems of equations separately one gets several equiprobable scenarios of matrices of utilizations for each k_2.

Theoretically, the moment fitting procedure, see Subsection 3.1, may be used again. However, its numerical tractability – numerical solution of a large nonconvex weighted least squares problem – is an open question.

The *scenario tuning procedure* by Popela (1998a) assumes that there are at disposal experts' scenarios $\hat{\boldsymbol{T}}^{k_2}$, $k_2 \in \mathcal{K}_2$. Hence, the corresponding number of second-stage decision vectors $\boldsymbol{x}_2^{k_2}$ that satisfy (3) for $k_2 \in \mathcal{K}_2$ is introduced. One expects that also the already recorded decisions, say, $(\hat{\boldsymbol{x}}^1, \hat{\boldsymbol{x}}^2)$ should be feasible, or nearly feasible, for each of experts' scenarios. If it is not the case, using the past recorded experience the input experts' scenarios are tuned for the sake of feasibility of the production process. One allows small perturbations $\boldsymbol{\Delta}^{k_2}$, $k_2 \in \mathcal{K}_2$ of the matrices of utilizations $\hat{\boldsymbol{T}}^{k_2}$ and applies the perturbations which provide the best fit. This can be done for by solving *separately* for each $k_2 \in \mathcal{K}_2$ the quadratic program

$$\text{minimize } \|\boldsymbol{\Delta}^{k_2}\|^2$$

with respect to $\Delta_{il}^{k_2}\ \forall i, l, k_2$, subject to

$$l_{i2} \ \leq \ \sum_{l=1}^{m_1}(\hat{\tau}_{il}^{k_2} + \Delta_{il}^{k_2}) \sum_{j \in J_1} a_{lj}\hat{x}_j^1 + \ \sum_{j \in J_2} a_{ij}\hat{x}_j^{k_2} \leq \ u_{i2},\ i = 1, \ldots, m_2$$

for all couples $(\hat{\boldsymbol{x}}^1, \hat{\boldsymbol{x}}^2)$.

The matrices $\boldsymbol{\Delta}^{k_2 *}$ of minimal perturbances are used to"tune" or update for each k_2 the initial experts' scenarios $\hat{\boldsymbol{T}}^{k_2}$ to $\hat{\boldsymbol{T}}^{k_2} + \boldsymbol{\Delta}^{k_2 *}$ which provide then the sought input for the stochastic program in question.

4 Model implementation

In this section, we illustrate discussions about melt control modeling principles introduced in previous sections by examples based on real-life data. We consider a steel production in one Czech foundry with the furnace capacity of 20 tons. We focus on a specific technology combining production steps realized in an electric-arc furnace (EAF) with a ladle furnace (LF) finishing. We restrict ourselves to the steel denoted 42CrMo. See Table 1 for the required composition of the produced alloy. The goal intervals $[l_{iT}, u_{iT}]\,\forall i$ are specified by decimal numbers between 0 and 1 defining how many kilograms of ith element should be contained in 1 kilogram of the produced liquid metal. The factory-defined goal intervals are tighter than those given by Czech standards to achieve the higher product quality. The required amount of the produced steel is 14.9 tons. The input materials, prices, their compositions, and elements are in Table 1. The whole production process is a multistage one. A two-stage model is applied first in Subsection 4.1. In this case, we may obtain it either by aggregation of all stages following the first stage or by modeling only the end of the production process. We begin with the second possibility. It may be interpreted as the model for the initial and final alloying phases. Frequently, the results have to be obtained under real-time control restrictions, so a sensible scenario generation becomes very important.

	c_j	t		l_{iT} and u_{iT}									
Material	Price	Stage	L/U	Fe	C	Mn	Si	P	S	Cr	Mo	Al	Cu
42CrMo		final	L	0.9500	0.0038	0.0060	0.0010			0.0100	0.0020	0.0001	
			U	0.9800	0.0050	0.0085	0.0035	0.0003	0.0003	0.0150	0.0028	0.0004	0.0001
Scrap	3.0	1	L	0.9600	0.0010	0.0040	0.0020	0.0001	0.0001	0.0020			0.0001
			U	0.9900	0.0040	0.0080	0.0040	0.0003	0.0004	0.0040			0.0003
RM955	3.9	1		0.9804	0.0025	0.0035		0.0003	0.0003	0.0110	0.0020		
T951	3.7	1		0.9829	0.0025	0.0035		0.0003	0.0003	0.0070	0.0035		
Coke	1.5	1 2			1.0000								
FeMo	525.0	2		0.3490	0.0010						0.6500		
FeSiMn	18.9	2 3		0.0850	0.0100	0.7000	0.2000			0.0050			
AlInput	50.0	2 3					0.0500					0.9500	
FeCr6	21.2	3		0.2350	0.0650					0.7000			
FeSi45	10.1	3		0.5497			0.4500	0.0002	0.0001				

Table 1: Produced alloy and input materials.

Then, in Subsection 4.2, a three-stage model adds the first stage called a charge to the previously discussed two-stage model. In addition, with the three-stage model, the interstage dependence and the random composition of the scrap must be considered.

The both subsections have a similar structure and use the same set of data. At first, available data are presented and discussed. Then, models already defined in Section 2 are slightly extended to allow realistic computations. The special attention is devoted to the collection of the model input data, and hence, to the generation of scenarios in different ways. Therefore, computations are realized for various deterministic reformulations starting from simple ones. The models are implemented in GAMS and solved using solvers implementing either standard (CPLEX and OSL) or decomposition-based solution algorithms (OSLSE). The most interesting results are further analysed.

4.1 Two-stage melt control

The electric-arc furnace with ladle teeming is chosen. The important general information can be found in Table 1, i.e. for $t = 1, 2, 3$ we find there sets of input materials, the number of considered elements $m = m_t = 10\,\forall t$, coefficients $c_j, j \in J_t$, a_{ij}, $j \in J_t, i = 1, \ldots, m$ and l_{it}, u_{it} $\forall i = 1, \ldots, m$. Specifically, we are interested now only in rows numbered by 2 and 3 in the Stage column of Table 1. To continue, we need more information about the process flow. It is important that all melts are documented in melt reports. Hence, together with general information contained in Table 1 that is valid for all technologies developed for 42CrMo steel production in combination of EAF and LF, we have several tables such as Table 2 storing information just about one melt. Tables 1 and 2 fully identify one run of the steel production process for 42CrMo. At first, the furnace is empty ($h_i^1 = 0\,\forall i$). During the charge stage the amounts of input materials (see the second column of Table 2(a) for x_j^1) must be taken into the account. Then, the mixture is melted and the composition is measured (multiply proportions and melt weight from the first column of Table 2(b) to obtain h_i^2). The alloying cost for the melt related with Table 2 is $z_{HB}^2 = 12351$ and the total melt cost equals $z_{HB}^1 = 70601$.

	x_j^t in kg		
Material	Stage 1	Stage 2	Stage 3
Scrap	5400		
RM955	6000		
T951	5000		
Coke	100	20	
FeMo		11	
FeSiMn		100	50
AlInput		8	2
FeCr6			130
FeSi45			45
Sum [kg]	16500	139	227

m	h_i^{T+1}/w^{T+1}		
Elements	Stage 1	Stage 2	Stage 3
Fe	0.9722	0.9634	0.9697
C	0.0029	0.0043	0.0049
Mn	0.0011	0.0056	0.0079
Si	0.0001	0.0001	0.0020
P	0.0002	0.0002	0.0002
S	0.0002	0.0002	0.0002
Cr	0.0066	0.0064	0.0124
Mo	0.0019	0.0024	0.0024
Al		0.0002	0.0003
Cu	0.0000	0.0000	0.0000
Melt weight w^{T+1} in kg	15299	14960	14928

Table 2: (a) Amounts of input materials in kg. (b) Measurements in 100%.

The idea may be applied repeatedly and at the end all x_j^t and h_i^{t+1} are specified. In addition, $h_i^{T+1} \in [l_{iT}, u_{iT}]$ so the required alloy is made. As in (5), we introduce $h_i^{k_{T+1}} = \sum_{l=1}^{m_T}(h_l^{a(k_{T+1})} + \sum_{j\in J_T} a_{lj} x_j^{a(k_{T+1})})$. For simplicity we replace $h_i^{k_t}$ by h_i^t when only one fixed scenario, completely specified by stage t and index i, is taken into account.

With the two-stage model, we are asked to continue the melting process, knowing the weight and chemical composition of the melt before alloying begins (see the first column of Table 2(b)). Although Tables 1 and 2 contain all necessary information for initial deterministic optimization, we still need to modify the model (1)–(4). We begin our computations in stage 2, i.e., after the charge and before alloying, having the first measurement results at disposal. Thus, we must recognize that the furnace already contains the melt. We also see that the composition of input materials for stages 2 and 3 is known. The goal intervals are specified by relative proportions, hence, l_{it}, and u_{it} represent fractions from unity. It is useful to introduce auxiliary variables $w^{k_t}, t = 1, \ldots, T+1$ denoting the amounts of melt at considered stages, specified as $w^{k_t} = \sum_{i=1}^{m_t} h_i^{k_t}$ for $t = 1, \ldots, T+1$ and $k_t \in \mathcal{K}_t$. Then, the bounds for *amounts* of elements are specified by $l_{it} w^{k_{t+1}}$ and $u_{it} w^{k_{t+1}}$. It is also necessary to define lower bound w_L^{T+1} for the minimum amount of the produced steel and the upper bounds $w_U^t, t = 1, \ldots, T+1$ derived from the furnace capacity. The updated two-stage model (1)-(4) is

$$\text{minimize} \sum_{j\in J_2} c_j x_j^2 \; + \sum_{k_3\in\mathcal{K}_3} p_{k_3} \sum_{j\in J_3} c_j x_j^{k_3} \tag{21}$$

subject to

$$l_{i2} w^{3E} \; \le \sum_{l=1}^{m_1} \tau_{il}^E (\; h_i^2 + \sum_{j\in J_2} a_{lj} x_j^2) \le \; u_{i2} w^{3E},\; i = 1, \ldots, m_2 \tag{22}$$

$$w^{3E} = \sum_{i=1}^{m_2} \sum_{l=1}^{m_1} \tau_{il}^E (\; h_i^2 + \sum_{j\in J_2} a_{lj} x_j^2) \tag{23}$$

$$l_{i3} w^{k_3} \; \le \; \sum_{l=1}^{m_2} \tau_{il}^{k_3} (h_l^2 + \sum_{j\in J_2} a_{lj} x_j^2) + \; \sum_{j\in J_3} a_{ij} x_j^{k_3} \le \; u_{i3} w^{k_3},\; i = 1, \ldots, m_3,\; k_3 \in \mathcal{K}_3 \tag{24}$$

$$w^{k_3} = \sum_{i=1}^{m_3}(\sum_{l=1}^{m_2} \tau_{il}^{k_3}(\ h_i^2 + \sum_{j\in J_2} a_{lj}x_j^2) + \sum_{j\in J_3} a_{ij}x_j^{k_3}), k_3 \in \mathcal{K}_3 \quad (25)$$

$$x_j^2 \geq 0,\ j \in J_2,\ x_j^{k_3} \geq 0,\ j \in J_3,\ k_3 \in \mathcal{K}_3,\ w^{k_3} \geq w_L^4,\ w^{k_t} \leq w_U^{t+1}, t = 2,3 \quad (26)$$

This is again a fixed recourse problem based on scenarios.

Before we may begin computations, we have to complete our input data set; see Table 3 for measurements h_i^2, bounds $l_{it}, u_{it}, t = 1,2,3$, and expert- and measurement-based utilizations $\hat{\tau}_{ii}^t, \tau_{ii}^t, i = 1,\ldots,m_t, t = 2,3$. (Notice the distinction between the expert-based utilizations $\hat{\tau}_{ii}^t$ and the standard or average utilizations τ_{il}^E introduced earlier.)

Scenario	Fe	C	Mn	Si	P	S	Cr	Mo	Al	Cu
$\hat{\tau}_{ij}^2$	0.94750	0.31750	0.25000	0.05950	0.60250	0.60250	0.88250	1.00000	0.00575	0.01750
τ_{ij}^2	0.94000	0.32000	0.24000	0.05600	0.60000	0.60000	0.88000	1.00000	0.00400	0.01000
h_i^2/w^2	0.97219	0.00291	0.00110	0.00006	0.00017	0.00018	0.00664	0.00190		0.00000
l_i^1		0.0018								
u_i^1	1.0000	0.0033	0.0038	1.0000	0.0004	0.0004	1.0000	0.0025	1.0000	0.0001
$\hat{\tau}_{ij}^3$	0.98300	1.00000	0.97625	0.04250	0.98750	0.98750	0.96000	1.00000	0.42625	1.00000
τ_{ij}^3	0.98500	1.00000	0.99500	0.04700	0.98000	0.97000	0.98000	1.00000	0.44000	1.00000
h_i^3/w^3	0.96336	0.00428	0.00558	0.00005	0.00017	0.00018	0.00644	0.00237	0.00022	0.00000
l_i^2		0.00350	0.00520	0.00005			0.00500	0.00200	0.00010	
u_i^2	1.00000	0.00440	0.00900	0.00350	0.00035	0.00035	0.01300	0.00280	0.00035	0.00010
l_i^3	0.95000	0.00380	0.00600	0.00100			0.01000	0.00200	0.00010	
u_i^3	0.98000	0.00500	0.00850	0.00350	0.00030	0.00030	0.01500	0.00280	0.00040	0.00010

Table 3: Melt composition, goal intervals, expert- and measurement-based utilizations.

Interstage goal intervals $l_{it}, u_{it} \forall i, t = 1,2$ have been derived by metallurgical rules, the experience of previous melts, and the goal interval relaxation. In foundry, $\tau_{ij}^{k_3}$ are often specified by experts. Usually, they consider only diagonal utilization matrices and keep in view one expert-based scenario, $\hat{\tau}_{ii}^t, i = 1,\ldots,m_t, t = 2,3$, for one stage. We may try to verify expert-based utilizations, e.g., for our two-stage model, computing $\hat{\boldsymbol{T}}^3(\boldsymbol{h}^2 + \boldsymbol{A}^2\boldsymbol{x}^2) + \boldsymbol{A}^3\boldsymbol{x}^3$ to get the final composition of melt. Boldface letters $\boldsymbol{x}^t, \boldsymbol{h}^t, \boldsymbol{A}^t$, and $\hat{\boldsymbol{T}}^t$ denote vectors and matrices having components x_j^t, h_i^t, a_{ij}^t, and $\hat{\tau}_{ij}^t$ respectively.

It can be found easily that in our example with the expert utilizations $\hat{\tau}_{ii}$, the goal requirements for 42CrMo are not satisfied (see Table 1) and even the final measurements cannot be obtained by the described matrix multiplications (cf. Table 2(b)). It seems that experts' suggestions are useless and even meaningless. This motivates a more careful scenario generation. By solving the separated system of linear equations (16) for unknown $\tau_{ii}^{k_3}$, with the known compositions a_{ij}, different amounts of inputs $x_j^{a(k_3)}$ and varying results of measurements $h_i^{a(k_3)}$ and $h_i^{k_3}$

$$h_i^{k_3} = \tau_{ii}^{k_3}(h_i^{a(k_3)} + \sum_{j\in J_2} a_{ij}x_j^{a(k_3)}),\ i = 1,\ldots,m_2,\ k_3 \in \mathcal{K}_3 \quad (27)$$

we obtain a measurement-based utilization τ_{ii}^3 which corresponds to one of scenarios $\tau_{ii}^{k_3}$. In a similar way, we get also measurement-based utilizations $\tau_{ii}^{k_2}$. In this way, we may trace realized trajectories of the considered production process. Table 4

presents four different scenarios of $\tau_{ii}^{k_2}$ (indexed by superscripts $k_2 = 1, \ldots, 4$) and four different scenarios $\tau_{ii}^{k_3}$ (indexed by superscripts $k_3 = 5, \ldots, 8$) derived from four melt reports, along with the related costs of the alloying and of the whole melting process.

Scenario	Fe	C	Mn	Si	P	S	Cr	Mo	Al	Cu	z_{HB}^t	$z_{SB_1}^t$
τ_{ii}^1	0.937	0.35	0.192	0.061	0.58	0.66	0.88	1.00	0.005	1.00	70610	55065
τ_{ii}^2	0.941	0.37	0.199	0.057	0.55	0.69	0.86	1.00	0.004	1.00	70040	56508
τ_{ii}^3	0.952	0.40	0.21	0.047	0.57	0.55	0.89	1.00	0.003	1.00	69359	54149
τ_{ii}^4	0.946	0.43	0.183	0.058	0.59	0.64	0.87	1.00	0.005	1.00	69326	57728
τ_{ii}^5	0.989	1.00	0.98	0.041	0.98	0.96	0.978	1.00	0.55	1.00	12351	4766
τ_{ii}^6	0.983	1.00	0.985	0.037	0.96	0.95	0.965	1.00	0.55	1.00	12725	5193
τ_{ii}^7	0.992	1.00	0.981	0.040	0.97	0.955	0.981	1.00	0.55	1.00	11680	4469
τ_{ii}^8	0.987	1.00	0.983	0.038	0.95	0.94	0.969	1.00	0.55	1.00	11466	5868

Table 4: Measurement-based scenarios of utilizations – sets $\mathcal{K}_2$ and $\mathcal{K}_3$.

Because our goal is to find another cheaper way to produce the same steel, we allow the 'computer knowing our two-stage program' to choose other inputs arbitrarily feasible but with the lowest cost; see Table 5 for results.

Inputs	$x_{j,HB}^2$	$x_{j,SB_1,\min}^2$	$\hat{x}_{j,SB_1,\min}^2$	$x_{j,SB_4,\min}^2$	$x_{j,SB_{4096},\min}^2$	$x_{j,SB_{cov},\min}^2$
Coke	20.0	7.0	6.8	16.9	17.6	17.3
FeMo	11.0	1.6	1.5	2.1	2.1	2.1
FeSiMn	100.0	87.5	89.4	105.0	107.7	106.0
AlInput	8.0	3.6	3.7	4.0	4.1	4.0
Objective function	z_{HB}^2 12351	$z_{SB_1,\min}^2$ 4766	$\hat{z}_{j,SB_1,\min}^2$ 4813	$z_{SB_4,\min}^2$ 6140	$z_{j,SB_{4096},\min}^2$ 7896	$z_{j,SB_{cov},\min}^2$ 6920

Table 5: Comparison of optimal solutions.

We also know the cost of the whole realized process and its part related to our two-stage problem, i.e. $z_{HB}^2 = 12351$ (with HB for 'history based' and superscript 2 referring to the current initial alloying second stage). It may be compared with one-scenario-based (deterministic) optimization solution $z_{SB_1,\min}^2 = 4766$ obtained from (21)-(26) model for $\mathcal{K}_3$ set having the only element. (The index SB_s denotes here and in the sequel scenario-based problems using s scenarios.) In this case, knowing losses in advance and utilizing our model, a melter could save more than 61% of alloying costs. This looks surprising, as the alloying stages could be considered as a source of minor changes regarding the initial charge. However, we must remember that we have obtained the best solution given a complete foresight, whereas in reality, losses are uncertain and unknown in advance. Therefore, we will apply now the true scenario-based stochastic programming model. At first, we assume that only a few melt reports for steel 42CrMo are available, namely, those which were used to get the four scenarios of utilizations $\tau_{ii}^{k_3}$, $k_3 = 5, \ldots, 8$ from Table 4. Assume that probabilities of scenarios are equal, so $p_5 = \ldots = p_8 = 1/4$. Using GAMS/OSL again to solve the program (21)-(26) with the new data set, we obtain $z_{SB_4,\min}^2 = 6140$ (savings 50.28%, see Table 5 for the optimal solution). We see that $z_{SB_1,\min}^2 \leq z^2{}_{SB_4,\min} \leq z^2{}_{HB,\min}$ in our example. However, computing the expected cost with respect to the four given scenarios for solution $\boldsymbol{x}_{SB_1,\min}^2$, i.e., checking first the feasibility in (21)-(26) and then computing the objective

function values for recourse actions $x_j^{k_3}$ obtained by the solution of separate linear programs, we get $z^2_{SB_4}(\boldsymbol{x}^2_{SB_1,\min}) = 9421$. This 23.72% saving in comparison with melter's decision (which was bad indeed in this case, but useful for illustration of our sceanrio-based approach) shows that even the deterministic approach may bring help in melt control and it illustrates why the scenario approaches are advantageous. We may also return back to expert-based utilizations $\hat{\tau}^t_{ii}$ which were marked as wrong for computations along individual scenarios. Using them for the two-stage problem at the place of the measurement-based τ^t_{ii} we get the related optimal value $\hat{z}^2_{SB_1,\min} = 4813$, less optimistic than $z^2_{SB_1}$ but the corresponding expected cost $z^2_{SB_4}(\hat{\boldsymbol{x}}^2_{SB_1,\min}) = 9024$ means saving of 26.93%. Hence, we can understand now why metallurgists are overestimating their expert-based estimates of utilizations. They intuitively use the worst case approach to avoid the surprise coming with the overfilled furnace. In certain sense, their aggregated values for $\hat{\tau}^t_{ii}$ are chosen to find a robust decision. Although we understand the basic idea, we must say that the reasoning is wrong, as we have a better scenario-based model.

At this moment, we have only four melt reports. A reasonable question is whether we may significantly increase the number of scenarios before more information about further melts is available. The simplest step is to assume independent random losses within the alloying stage. Therefore, we may create new scenarios easily just combining all elements utilizations. The number of derived scenarios becomes large, equals $4^6 = 4096$ as four utilizations remain constant, independent of the scenario changes; see Table 4. Several solvers (CPLEX, OSL, OSLSE) have been tested with the GAMS source code. The results are the same, we obtain $z_{SB_{4096},\min} = 7896$, however different amounts of computing time were needed. In this case, we can see even from the visual analysis of scenarios in Table 4 that there might be some nonzero correlations among utilizations. Thus, our instage-independence-based scenario set is built in a too defensive way: We skipped an available information, hence, we considered also unrealistic scenarios and their recourse costs were taken into account. As a result we obtained a too pessimistic solution. Still, the obtained solution may be implemented and used as it gives a good chance to decrease the melting costs. However, there is a bottleneck. This approach is useless for a large set of scenarios because computations for the alloying stage should be realized in real time, i.e. at most during tens of seconds. For this purpose, scenario set reduction techniques have been developed (see Popela and Zeman (1999) for the application of principal components and Popela and Roupec (1999) for identification of so called extreme scenario sets). In this paper we try to remove the unrealistic independence assumption and to apply another approach – the moment fitting procedure from Subsection 3.1. As the number of scenarios is quite small (4 melt reports till now) to obtain reasonable values for covariances, we may exploit general metallurgical laws and experience. The idea is that the relationships among utilizations are quite general and they do not vary too much when similar steels are produced and similar technologies are used. Therefore, we set the values of ρ_{il} analyzing similar steel melt reports, we roughly estimate μ_i and σ_i^2 from 42CrMo steel melt reports. We have 8 new scenarios minimizing the objective

(17) from Subsection 3.1 (with equal weights) under additional experience-based constraints saying that scenarios have nearly the same probabilities and that the utilizations are bounded below and above by the existing extremal cases of their values derived from melt reports. The obtained scenarios are listed in Table 6 and the optimal value is 6920; see the last column of Table 5.

Scenario	Fe	C	Mn	Si	P	S	Cr	Mo	Al	Cu	p_k
τ_{ij}^1	0.989	1.00	0.98	0.041	0.98	0.956	0.968	1.00	0.55	1.00	0.628
τ_{ij}^2	0.983	1.00	0.985	0.037	0.96	0.945	0.965	1.00	0.55	1.00	0.623
τ_{ij}^3	0.992	1.00	0.981	0.040	0.97	0.952	0.971	1.00	0.55	1.00	0.620
τ_{ij}^4	0.987	1.00	0.983	0.038	0.95	0.949	0.969	1.00	0.55	1.00	0.630
τ_{ij}^5	0.988	1.00	0.982	0.042	0.98	0.961	0.978	1.00	0.55	1.00	0.626
τ_{ij}^6	0.984	1.00	0.984	0.039	0.95	0.953	0.975	1.00	0.55	1.00	0.625
τ_{ij}^7	0.993	1.00	0.981	0.040	0.97	0.955	0.981	1.00	0.55	1.00	0.626
τ_{ij}^8	0.987	1.00	0.983	0.037	0.96	0.944	0.979	1.00	0.55	1.00	0.622

Table 6: Fitted scenarios and probabilities.

At this moment we may think what to do when more melt reports are available. At the first glance, it seems that there is no necessity to build artificial scenarios as the number of scenarios is growing, they are related to historical melts and they may be considered as representative enough. However, after one year with several melts per day we face again the question how to reduce the number of scenarios under real-time restrictions. Hence, the approaches of Subsection 3.1 and those suggested in Popela and Zeman (1999) or Popela and Roupec (1999) remain useful. The list may be completed by methods based on random sampling from the given huge set of scenarios, see e.g. Shapiro and Homem-de-Mello (1998).

4.2 Three-stage melt control

As the next step we continue with a three stage model: We want to include optimization of the first step of 42CrMo production. One possibility is reduction of the number of stages to two simply thinking about all alloying steps incorporated in one stage only. In this case, we optimize the charge under rather rough forecasting of all further alloying consequences. This is definitely better than one stage model and it is comparable with another approximating two-stage model whose horizon is restricted to the end of the first alloying stage. As we have discussed before, because the charge can be computed in advance, we are not restricted by the real-time requirements and our model my be larger. Again we use the update of the previously introduced model (5)–(10) incorporating melt weights w^{k_t} and relative goal intervals l_{it} and u_{it}. At the beginning, we assume that the scrap composition is fixed to the midpoints of intervals.

The resulting form of the model is

$$\text{minimize} \sum_{j \in J_1} c_j x_j^1 \; + \sum_{t=2}^{3} \sum_{k_t \in \mathcal{K}_t} p_{k_t} \sum_{j \in J_t} c_j x_j^{k_t} \tag{28}$$

subject to

$$\sum_{l=1}^{m_{t-1}} \tau_{il}^{k_t} \left(h_l^{a(k_t)} + \sum_{j \in J_{t-1}} a_{lj} x_j^{a(k_t)} \right) - h_i^{k_t} = 0,\ i = 1, \ldots, m_{t-1},\ k_t \in \mathcal{K}_t,\ t = 2, 3 \tag{29}$$

$$l_{it-1} \sum_{k_t \in \mathcal{K}_t} p_{k_t} w^{k_t} \leq \sum_{k_t \in \mathcal{K}_t} p_{k_t} h_i^{k_t} \leq u_{it-1} \sum_{k_t \in \mathcal{K}_t} p_{k_t} w^{k_t},\ i = 1, \ldots, m_{t-1},\ t = 2, 3 \tag{30}$$

$$l_{i3} w^{k_4} \leq h_i^{k_3} + \sum_{j \in J_3} a_{ij} x_j^{k_3} \leq u_{i3} w^{k_4},\ i = 1, \ldots, m_3 \tag{31}$$

$$w^{k_t} = \sum_{i=1}^{m_t} h_i^{k_t},\ k_t \in \mathcal{K}_t,\ t = 1, \ldots, 4 \tag{32}$$

$$(x_1^1, \ldots, x_{n_1}^1) \in \mathcal{X},\ x_j^{k_t} \geq 0,\ k_t \in \mathcal{K}_t,\ j \in J_t,\ t = 2, 3 \tag{33}$$

$$w^{k_3} \geq w_L^4,\ w^{k_t} \leq w_U^{t+1}, t = 1, 2, 3. \tag{34}$$

With the charge, special metallurgical constraints may be added, from the simplest bounds on the scrap amount, on the sum of amounts of return materials to the additional constraints based on linear functions of the melt composition (restricted amount of the sum of phosphorus and sulfur, satisfactory C-equivalent, etc., see Popela (1998b)). In general, all these constraints can be modeled by a polyhedral set $\mathcal{X}$. Therefore, we do not present them explicitly although they were utilized in computations.

We are ready now to present computational results. The input data are again taken from Tables 1–4. At first, we get $z^1_{HB,\min} = 70601$ as a consequence of the melt report information. Then, knowing melt related inputs $x_j^t, j \in J_t, t = 2, 3$ and measurement results $h_i^t, i = 1, \ldots, m_t, t = 2, 3$, solving two systems of separate linear equations (16), we can easily identify the measurement-based scenario of utilizations related with the melt report in question, which consists of *two* diagonal matrices with elements $\tau_{ii}^{k_t}, i = 1, \ldots, m_t, t = 2, 3$. In the sequel, we solve program (28)–(34) for this scenario and obtain the best single scenario cost $z^1_{SB_1,\min} = 55065$. Similarly, we may obtain results for remaining three scenarios. The last two columns of Table 4 indicate that we may save between $16 - 22\%$. However, further computations provide Table 7 that is based on analogous analysis as Table 5 for two-stage model. And the conclusion is quite clear, real savings with just one-scenario approach will be significantly lower. Therefore, we utilize approaches developed in Section 3. Using the four melt reports we may derive data

Inputs	$x^1_{j,HB}$	$x^1_{j,SB_1,\min}$	$\hat{x}^1_{j,SB_1,\min}$	$x^1_{j,SB_4,\min}$	$x^1_{j,SB_{16},\min}$	$x^1_{j,SB_{cov},\min}$	$x^1_{j,SB_{ri,cov},\min}$
RM955	6000.0	5780.3	5830.6	5902.1	6172.6	5930.3	6010.6
T951	5000.0	4900.5	4930.5	4950.1	5120.1	5200.1	5150.5
Scrap	5400.0	5600.2	5809.7	5510.0	5107.7	4960.0	4820.3
Coke	210.0	180.4	200.3	190.0	198.1	192.0	193.0
Objective function	z^1_{HB} 70601	$z^1_{SB_1,\min}$ 55065	$\hat{z}^1_{j,SB_1,\min}$ 57112	$z^1_{SB_4,\min}$ 59601	$z^1_{j,SB_{16},\min}$ 61396	$z^1_{j,SB_{cov},\min}$ 60020	$z^1_{j,SB_{ri,cov},\min}$ 60997

Table 7: Comparison of optimal solutions.

for 4-scenario-based model. Exploitation of these melt reports on the complete melt implies that there is a one-to-one correspondence of the terminal nodes indexed by $5,\ldots,8$ and their ancestors, $a(k) = k-4$, $k = 5,\ldots,8$, elements of $\mathcal{K}_2$ listed in Table 4. The computation based on the fan of these four scenarios will return the total expected cost of the production process (consisting now of the charge and alloying) $z^1_{SB_4,\min} = 59601$. As in Subsection 4.1, we may compute values $z^1_{SB_4}(\boldsymbol{x}^1_{HB}) = 66934$ for the historical first-stage solution, $z^1_{SB_4}(\boldsymbol{x}^1_{SB_1,\min}) = 63121$ for the optimal first-stage solution based on one scenario and $z^1_{SB_4}(\hat{\boldsymbol{x}}^1_{SB_1}) = 62115$ for the optimal first-stage solution based on one scenario of expert-based utilizations. For the related first-stage optimal solutions $\boldsymbol{x}^1$ and some additional results see Table 7.

The conclusion is similar to that in 4.1. The use of optimization, even for the deterministic (one-scenario) problem is better then the melter's intuition and experience (9.18% savings in our example). The practitioners may achieve some improvement using the expert-based scenario (10.18% savings) to hedge against uncertainty. Nevertheless, the numerical results provide again an evidence that the proposed scenario-based stochastic programming approach is the right way to hedge against uncertainty (15.58% savings for four scenarios).

The next questions are: 'How to increase the number of scenarios and how to create a nontrivial scenario tree to capture the multistage decision structure of the problem?' The first step could be simple. As discussed in Subsection 3.2, we may assume the interstage independence. Accordingly, we may generate a scenario tree with $4^2 = 16$ scenarios from Table 4. The result is $z^1_{SB_{16,\min}} = 61396$ (12.42% savings).

In practise, the interstage dependence has not yet been analyzed, so it is hard to get reliable experts' opinions. The common sense says that if random utilizations of certain elements have been high for the first stage, it may be caused by a short time of heating and the non-realized loss may be realized during the alloying phase. Hence, negative correlations between stage related utilizations may occur. We shall apply again the general idea of moment fitting approach assuming for simplicity that the instage structure of utilizations is fully specified by the related melt report and also that there is no dependence between losses of different elements belonging to different stages. Similarly as in Subsection 4.1 we use the moment conditions on utilizations in both stages and on their interstage correlations. In the fitting objective function, such as (17), we put a lower weight (25%) on terms fitting interstage covariances. The results of our computations are listed in Table 8. The last but one column of Table 7 indicates that some improvement of the objective function value may be expected. The question is the reliability of our estimates on interstage dependence characteristics.

The next step is to consider the random input. We know from Table 1, that this concerns only the scrap. We may extend our model (28)–(34) adding first stage input scenarios using a modification of (11)–(14) to the three stage problem.

We consider only random content of chromium (Cr) as it is the important element for the final composition and iron (Fe) as it is the main part of the alloy

Symbol	Fe	C	Mn	Si	P	S	Cr	Mo	Al	Cu	p_k
τ_{ij}^1	0.981	1.00	0.978	0.040	0.99	0.96	0.961	1.00	0.55	1.00	0.272
τ_{ij}^2	0.978	1.00	0.985	0.037	0.96	0.945	0.965	1.00	0.55	1.00	0.230
τ_{ij}^3	0.989	1.00	0.979	0.040	0.97	0.952	0.971	1.00	0.55	1.00	0.210
τ_{ij}^4	0.982	1.00	0.983	0.038	0.95	0.949	0.969	1.00	0.55	1.00	0.288
τ_{ij}^5	0.988	1.00	0.982	0.042	0.98	0.956	0.977	1.00	0.55	1.00	0.526
τ_{ij}^6	0.984	1.00	0.978	0.039	0.94	0.951	0.975	1.00	0.55	1.00	0.474
τ_{ij}^7	0.991	1.00	0.984	0.041	0.97	0.952	0.978	1.00	0.55	1.00	0.591
τ_{ij}^8	0.987	1.00	0.987	0.038	0.96	0.947	0.976	1.00	0.55	1.00	0.409
τ_{ij}^9	0.982	1.00	0.981	0.042	0.98	0.961	0.978	1.00	0.55	1.00	0.544
τ_{ij}^{10}	0.985	1.00	0.986	0.040	0.97	0.953	0.975	1.00	0.55	1.00	0.456
τ_{ij}^{11}	0.993	1.00	0.982	0.041	0.97	0.955	0.981	1.00	0.55	1.00	0.601
τ_{ij}^{12}	0.987	1.00	0.985	0.035	0.96	0.944	0.979	1.00	0.55	1.00	0.399

Table 8: Fitted scenarios and probabilities for 4x2 tree.

and it indirectly describes the variation of the remaining elements within the scrap. We create $3^2 = 9$ equiprobable scenarios for scrap composition using ideas of Subsection 3.3 (scenarios specified by $a_{ij}^E - \delta_{ij}$, a_{ij}^E, $a_{ij}^E + \delta_{ij}$, with $\delta_{ij} = 2/3(\overline{a}_{ij} - a_{ij}^E)$). Finally, we combine these 9 scenarios with 8 scenarios of utilizations obtained by the moment fitting procedure listed in Table 8. The scrap composition data are utilized from Table 1. Bounds $\underline{a}_{ij}$ and $\overline{a}_{ij}$ are given by the related rows denoted by L and U. The choice of the coefficient 2/3 was preferred because it generates more distinct scenarios than other possibilities.

Analyzing results from Table 7, we may conclude that considering the random composition of the scrap we have significantly decreased the risk of its use (cf. inputs for other scenario-based models and notice the difference in the scrap input).

The obtained results based on 72 scenarios ($z^1_{SB_{ri,cov,\min}} = 60997$ and savings 13.60%, see the last column of Table 7) are considered as the most realistic and numerically tractable representation of uncertainty achieved so far.

We emphasize that for illustrating purposes, we have chosen several melt reports of a beginning melter. Because he was significantly unsuccessful, we have got the opportunity to explain many different aspects just with one data set. Although data from experienced melters are 'more boring', numerical experience shows that also in these cases, we may save significant amount of money. The one-scenario models are usually forecasting savings about 12% whereas the real savings decrease to 3%. With scenario models discussed above it is quite realistic to expect real savings between 5% and 10%.

Regarding the modeled technological process, the obtained solutions must be interpreted as suboptimal only. Nevertheless, they are significantly better than solutions obtained by the contemporary techniques used in melt control.

In this context there is also an interesting interpretation of EVSI (Expected Value of Scenario Information - see Birge and Louveaux (1997)) for the charge problem: We may compare whether the costs of sorting scrap are less than the attained profit. It means to compute the model for unsorted scrap and compare the results for the case when the scrap composition is fully known. In our case it means to distinguish different types of input scraps with small intervals of uncertainty for a_{ij}.

5 Discussion and extensions

The scope of this paper has been mostly restricted to the iron production problem without inclusion of technological and storage constraints. Neither the environmental aspects nor the quality of production were taken into account. In principle, without any problems the model may be extended for additional deterministic constraints and, following ideas of multi-objective decision making, the objective function may be augmented for an additional term related to the pollution limitations or to the metal quality.

The models and their implementation were based on several assumptions, such as diagonal utilization matrices and fully specified input composition, interstage independence of utilities, their independence of the input quantities. Some of these assumptions were relaxed in the discussed scenario generation procedure which in turn made use not only of historical measurements, but exploited also experts' opinion. Similarly as for other applications of scenario-based stochastic programs, no recipe for the best scenario generation and selection procedure exists. We observed that simple, sound discretization procedures gave different scenarios. Sensitivity of the results (of the minimal costs and the best initial charge) with respect to the selected scenarios and their robustness related to various simplifying assumptions should be carefully analyzed. Bounds based on the best- and worst case analysis delineated briefly in Subsection 3.3 may help.

At the end, three specific problem and model properties of the stochastic program in question have to be underlined:
1. Stages are not defined by modeler's choice because they are given by the modeled production process;
2. Because the filled furnace cannot be enlarged or emptied during the process (contrary to the assumed unlimited borrowing and lending possibilities in financial applications, e.g.), the related hard constraints imply that relatively complete recourse cannot be assumed. Hence, feasibility of the first-stage solution must be analyzed; and
3. Computations related to the alloying stages should be realized in real time and this asks for a numerically tractable scenario generation procedure which results in a relatively small number of representative scenarios.

Acknowledgements. This work was partly supported by research projects *Mathematical Methods in Stochastics* – MSM 113200008 and MSMT CEZ: J22/98: 261100009 and by the Grant Agency of the Czech Republic (grants 106/01/1464, 201/02/0621 and 402/02/1015).

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